An Exposé of Trapped Electron Effects in the Plasma Expansion Problem (unpublished)

A current plasma expansion model reveals an electron equation of state in which electrons are confined by the electrostatic potential $\Phi(x)$ for which we assume for the moment $-\Psi \leq \Phi \leq 0$ with a finite $\Psi > 0$. As shown e.g. in Schamel's 2000 paper and in the literature cited therein (H.Schamel, Phys. Plasmas 7 4831 (2000)) the appropriate density expression in case of a negative potential becomes for isothermal electrons in a thermal plasma ($\beta = 1$)

$$n_e(\Phi) = A'(I(\Phi + \Psi) + e^{\Phi+\Psi} erf(\sqrt{\Phi + \Psi})) \equiv A'e^{\Phi+\Psi}, \qquad (1)$$

where $I(x)$ is defined by $I(x) = e^x(1 - erf\sqrt{x})$ and A' is a constant. The first part in (1) represents the free, the second part the trapped electrons. The factor A' is determined by the requirement $n_e(0) = 1$ and becomes

$$A' = \frac{1}{I(\Psi)+e^{\Psi} erf(\sqrt{\Psi})} \equiv e^{-\Psi}, \qquad (2)$$

Of course, a much simpler expression would be $n_e(\Phi) = e^{\Phi}$, but by writing n_e this way one can easily extend it to $\beta < 1$ and discuss furthermore the relationship between free and trapped electrons.

From (1) with (2) or from the Botzmann law it is easily seen that the density at potential minimum $-\Psi$ tends to zero as $\Psi \to \infty$ as it should for an expansion into a vacuum. Moreover, the first part in (1) vanishes in this limit for any Φ since $erf(\sqrt{\Phi + \Psi}) \to 1$ for $\Psi \to \infty$. The whole density is hence given by that of the trapped particles namely by the second term.

If we now allow for $0 < \beta \leq 1$, the difference will be that the second term in (1) is replaced by $\frac{1}{\sqrt{\beta}} e^{\beta(\Phi+\Psi)} erf\sqrt{\beta(\Phi + \Psi)}$ and accordingly in (2). From which follows immediately that in the infinite Ψ limit, when the first term again vanishes, the electron density generally becomes

$$n_e(\Phi; \beta) = e^{\beta\Phi} \qquad (3)$$

being again composed only of trapped electrons.

But this implies that if we take the actual electron temperature, given by that of the trapped electrons, $T_{et} = \frac{T_{ef}}{\beta}$, and change the normalization of Φ by using T_{et} instead of T_{ef} we return to the old isothermal equation of state. Schamel's trapping model in a correct representation hence does not give rise to a new situation as long as free electrons are missing corresponding to an infinite potential depth ansatz.

In this situation the electron front, i.e. the location where the electrons are adapted to the vacuum, is at infinity at any instant. On the other hand, to have an electron front x_{ef} at finite space this would require a singular potential, $\Psi \to \infty$, at x_{ef} beyond which there would be an exact vacuum. A

removal of this nonphysical aspect, however, means that the assumption of inertialess electrons has to be given up in favor of a time-dependent electron dynamics to be described in phase space.
HS 31.12.2012